

Exercise 47

Find the horizontal and vertical asymptotes of each curve. If you have a graphing device, check your work by graphing the curve and estimating the asymptotes.

$$y = \frac{5 + 4x}{x + 3}$$

Solution

Calculate the limits as $x \rightarrow \pm\infty$ to determine the horizontal asymptote. In the second limit, make the substitution, $x = -u$, so that as $x \rightarrow -\infty$, $u \rightarrow \infty$.

$$\lim_{x \rightarrow \infty} \frac{5 + 4x}{x + 3} = \lim_{x \rightarrow \infty} \frac{\frac{5}{x} + 4}{1 + \frac{3}{x}} = \frac{0 + 4}{1 + 0} = 4$$

$$\lim_{x \rightarrow -\infty} \frac{5 + 4x}{x + 3} = \lim_{u \rightarrow \infty} \frac{5 + 4(-u)}{(-u) + 3} = \lim_{u \rightarrow \infty} \frac{\frac{5}{u} - 4}{-1 + \frac{3}{u}} = \frac{0 - 4}{-1 + 0} = 4$$

Therefore, the horizontal asymptote is $y = 4$. The vertical asymptote is found by setting what's in the denominator equal to zero and solving for x .

$$x + 3 = 0$$

$$x = -3$$

